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5 SEM TDC PHY M 1

2016

(November)

PHYSICS

(Major)

Course : 501

(Mathematical Physics)

Full Marks : 60

Pass Marks : 24 (Backlog)/18 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct option : 1×6=6

(a) Which of the following integrals is non-vanishing?

(i) $\int_{-1}^{+1} xP_n \frac{dP_m}{dx} dx$ for $n > m$

(ii) $\int_{-1}^{+1} P_n(x)\,dx$

(iii) $\int_{-1}^{+1} x^2 P_5(x)\,dx$

(iv) $\int_{-1}^{+1} P_0(x)\,dx$

(b) Given $\Gamma(3)\Gamma\left(\frac{5}{2}\right) = A\Gamma(6)$, find A.

(i) $\sqrt{\pi}$

(ii) $\sqrt{\pi}/2$

(iii) $\sqrt{\pi}/2^3$

(iv) $\sqrt{\pi}/2^5$

(c) If $u = x^3 - 3xy^2$, the analytic function $f(z) = u + iv$ will be

(i) z^3

(ii) z^{-3}

(iii) $|z|^3$

(iv) None of the above

(d) What is the ratio of coefficients of z^n and $\frac{1}{z^n}$ in the Laurent's expansion of the function $\cosh\left(z + \frac{1}{z}\right)$?

(i) 0

(ii) $\frac{1}{2}$

(iii) 1

(iv) None of the above

(e) The value of a_0 in the Fourier series of t^2 in the interval $-\pi < t < \pi$ is

(i) 0

(ii) $\frac{\pi^2}{3}$

(iii) $\frac{\pi^2}{8}$

(iv) $\frac{\pi^2}{4}$

(f) Using Fourier integral, the value of $\int_0^\infty \frac{\cos xu}{1+u^2} du$ $(x > 0)$ is found to be

(i) $\frac{\pi}{2}$

(ii) $\frac{\pi}{2} e^x$

(iii) $\frac{2}{\pi} e^{-x}$

(iv) $\frac{\pi}{2} e^{-x}$

2. (a) Prove that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$$ 2

(b) Find the value of a_n in the Fourier series of $f(x)$ in the interval $(-\pi, \pi)$, where

$$f(x) = \pi + x, \text{ when } -\pi < x < 0$$
$$= \pi - x, \text{ when } 0 < x < \pi$$ 2

(c) Prove that $P_{2m}(-\mu) = P_{2m}(\mu)$. 2

(d) Express the integral $I = \int_0^\infty \frac{x^3}{(1+x)^5} dx$ in terms of beta and gamma functions and hence find its value. 2

(e) Using Cauchy's integral formula, evaluate the integral $\oint \frac{z^2}{(z^2-1)} dz$ around the unit circle with centre at $z = 1$. 2

(f) If $u(x, y) = x^2 - y^2$ is the real part of an analytic function $f(z) = u + iv$, find v. 2

3. (a) Solve the equation $y'' - y = 0$ with $y(0) = 4$, $y'(0) = -2$. 3

(b) Find the solution of the non-homogeneous equation $y'' + 4y = 8x^2$. 3

(c) Prove that

$$P_n(x) = \frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n (x^2 - 1)^n$$ 4

(d) Prove that

$$(2n+1)\,xP_n(x) = (n+1)\,P_{n+1}(x) + nP_{n-1}(x) \qquad 4$$

Or

Prove that Legendre polynomial $P_n(\mu)$ is the coefficient of h^2 in $(1-2\mu h+h^2)^{-1/2}$. 4

4. (a) Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then u and v satisfy $\nabla^2 u = 0$ and $\nabla^2 v = 0$. 4

(b) Prove that if $f(z)$ is an analytic function on and within the closed contour c, the value of $f(z)$ at a point $z = \varepsilon$ inside c is given by

$$f(\varepsilon) = \frac{1}{2\pi i}\oint \frac{f(z)}{z-\varepsilon}\,dz \qquad 4$$

(c) Answer any *two* from the following : 3×2=6

(i) Show that the triangle whose vertices are the points z_1, z_2, z_3 in Argand diagram will be equilateral if

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

(ii) If $f(z)$ is an analytic function of $|z|$, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$

(iii) Obtain the expansion

$$f(z) = f(a) + 2\left\{\frac{z-a}{2} f'\left(\frac{z+a}{2}\right) + \frac{(z-a)^3}{2^3\,3!} f'''\left(\frac{z+a}{2}\right) + \frac{(z-a)^5}{2^5\,5!} f^{(5)}\left(\frac{z+a}{2}\right) + \cdots\right\}$$

and determine its range of validity.

5. *(a)* Find an even function of x which is equal to kx for $0 \le x \le l/2$ and is

$$k(l-x) \text{ for } l/2 \le x \le l$$ 3

(b) Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval $-\pi < x < \pi$, where

$$f(x) = 0 \quad , \quad \text{when } -\pi < x < 0$$
$$= \frac{\pi x}{4} , \quad \text{when } 0 < x < \pi$$ 4

(c) Show that the rectified current through a half-wave rectifier is

$$I(t) = \frac{I_0}{\pi} - \frac{2I_0}{\pi}\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \cdots\right) + \frac{1}{2} I_0 \sin \omega t$$ 4

(d) State and prove Parseval's theorem. 3

Or

Obtain the Fourier series for a triangular wave given by

$y = 0$ at $t = 0$

$y = a$ at $t = T/2$

$y = 0$ at $t = T$ 3

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5 SEM TDC PHY M 2

2016

(November)

PHYSICS

(Major)

Course : 502

(Electrodynamics)

Full Marks : 60

Pass Marks : 24 (Backlog) / 18 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×6=6

(a) Which of the following relations is correct?

(i) $\sqrt{\varepsilon_0 E_0} = \sqrt{\mu_0}$

(ii) $E_0 = \sqrt{(\varepsilon_0 \mu_0)} B_0$

(iii) $\sqrt{(\varepsilon_0 \mu_0)} E_0 = B_0$

(iv) $\sqrt{\varepsilon_0 E_0} = \sqrt{\mu_0 B_0}$

(b) In polarization for normal incidence, the reflected coefficient (R) and transmission coefficient (T) is related by

(i) $R + T = 1$

(ii) $R + T = 2$

(iii) $R = T$

(iv) $R = 2T$

(c) If V is the potential difference between the two ends of a wire of length L, the magnetic field is circumferential at the surface of radius r, then the magnitude of the Poynting vector is

(i) $\frac{VI}{2\pi rL}$

(ii) $\frac{VI}{4\pi rL}$

(iii) $\frac{2VI}{\pi r^2 L}$

(iv) $\frac{2}{3}\frac{VI}{\pi rL}$

(Where the symbols have their usual meanings.)

(d) The kinetic energy of a particle moving with relativistic speed v is given by

(i) $\frac{1}{2}mv^2$

(ii) $\frac{1}{2}\frac{m_0 v^2}{\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}}$

(iii) $\frac{m_0}{\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}}c^2$

(iv) $\left(\frac{m_0}{\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}}-m_0\right)c^2$

(Where the symbols have their usual meanings.)

(e) In electric and magnetic field vectors of a monochromatic plane wave in conducting medium, the skin depth is determined by the relation

(i) $\left(\frac{2\omega}{\mu_0\sigma}\right)^{1/2}$

(ii) $\left(\frac{2}{\mu_0 \sigma \omega}\right)^{1/2}$

(iii) $\left(\frac{\sigma}{2\mu_0 \omega}\right)^{1/2}$

(iv) $\left(\frac{\sigma \omega}{2\mu_0}\right)^{1/2}$

(Where the symbols have their usual meanings.)

(f) The total power radiated by an accelerated charge at low velocity is

(i) $\frac{3}{2}\frac{e^2a^2}{\pi\varepsilon_0\varepsilon}$

(ii) $\frac{e^2a^2}{2\pi\varepsilon_0\varepsilon}$

(iii) $\frac{e^2a^2}{4\pi\varepsilon_0\varepsilon}$

(iv) $\frac{e^2a^2}{6\pi\varepsilon_0\varepsilon^3}$

(Where the symbols have their usual meanings.)

2. Answer any *five* of the following : 3×5=15

(a) What are the various properties of electromagnetic wave?

(b) Establish Maxwell's first equation in differential and integral forms.

(c) A neutron is travelling through the laboratory at three-fifths of speed of light. If the lifetime of neutron is 16 min, how long does it last?

(d) Derive and explain Brewster's law on the basis of electromagnetic theory.

(e) Discuss the phenomenon of total internal reflection of electromagnetic waves.

(f) Explain in brief the invalidity of ether hypothesis.

(g) Deduce the differential form of Lorentz gauge.

3. How was displacement current in electromagnetic wave introduced by Maxwell from generalized Ampere's law? 5

4. Find the momentum density and radiation pressure of electromagnetic waves. 2+2=4

5. Deduce the equation for electric and magnetic field vectors of electromagnetic waves propagated in a conducting medium. 5

6. Deduce Fresnel's equation for reflection and refraction of electromagnetic wave at normal incidence. 5

7. How is the polarization of an electromagnetic wave affected when it crosses the plane interface between two dielectrics? 5

8. Calculate the time averaged energy density of an electromagnetic wave in a conducting medium. 5

Or

Derive the equation for phase velocity of electromagnetic wave propagating in conducting medium. 5

9. Derive Lorentz transformation equations. 5

10. *(a)* Deduce Einstein mass energy relation $E = mc^2$. 3

(b) What do you mean by improper length? 2

Or

Derive the relation for the relativistic transformation of velocities. 2

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5 SEM TDC PHY M 3

2016

(November)

PHYSICS

(Major)

Course : 503

(Atomic and Molecular Physics)

Full Marks : 60

Pass Marks : 24 (Backlog) / 18 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Fill in the blanks (any *five*) : 1×5=5

(a) The value of spin quantum number of an electron in hydrogen atom is _____.

(b) The *D*-lines of sodium originate from _____ transition.

(c) Normal Zeeman effect occurs only in atoms which have a total spin *S* equal to _____.

(d) For heavier atoms _____ coupling holds.

(e) The procedure to achieve population inversion is called _____.

(f) If $\Delta\nu = \frac{eB}{4\pi m}$ is the frequency shift in the Zeeman splitting of a spectrum, then the corresponding wavelength shift $\Delta\lambda =$ _____.

2. Answer any *five* of the following : 2×5=10

(a) Find the possible values of j and m_j for states $l=3$ and $s=\frac{1}{2}$.

(b) State Bohr's postulates regarding the atomic model.

(c) Calculate the radius of the first Bohr orbit of hydrogen atom. Given $e=1{\cdot}6\times10^{-19}$C, $h=6{\cdot}63\times10^{-34}$ joule-sec, $k=9\times10^{9}\,\mathrm{N\,m^2/c^2}$ and $m=9{\cdot}1\times10^{-31}$kg.

(d) Discuss the essential requirements for producing laser action.

(e) Distinguish between Raman scattering and Rayleigh scattering.

(f) Calculate Lande's g-factor for s-electron.

3. *(a)* Describe the different types of coupling in atom. 5

Or

The first member of Balmer series of hydrogen has a wavelength of 6563 Å. Calculate the wavelength of its *(i)* second order and *(ii)* third order. In which region of the e.m. spectrum does this series lie? 2+2+1=5

(b) Describe the principle, construction and working of Ruby laser with necessary diagram. 5

4. Discuss the Sommerfeld theory of elliptical orbit of hydrogen atom and compare its results with those of Bohr's theory of circular orbits. 7

Or

What is Raman effect? Prove that to be Raman active, a molecular vibration or rotation must cause some change in molecular polarizability. Explain Raman lines intensity or polarization states from classical theory. 1+5+1=7

5. What are Stokes and anti-Stokes lines? In an experiment, the exciting line is at $\lambda = 5460$ Å and the Stokes line is at $\lambda = 5520$ Å. Find Raman shift and wavelength corresponding to anti-Stokes line. 2+5=7

Or

Discuss vibrational-rotational spectra of diatomic molecules with energy-level diagram. What are P and R branches in vibrational-rotational spectra? 5+2=7

6. What is anomalous Zeeman effect? In a normal Zeeman experiment, the Ca 4226 Å line splits into three lines separated by 0·25 Å in a magnetic field of 3 T. Determine e/m for the electron from these data. 2+4=6

Or

What are the drawbacks of Rutherford's atomic model? Discuss briefly the success and failure of Bohr's atomic model. 2+2+2=6

7. Derive an expression for magnetic moment of orbiting electron. Why is orbital magnetic momentum (μ_l) oppositely directed to orbital angular momentum (P_l)? 5+1=6

Or

What is Larmor precession? An atomic dipole is subjected to very strong magnetic field B so that it begins to precess about the field. Calculate the frequency of Larmor precession. 1+5=6

8. Write short notes on (any *three*) : 3×3=9
   (a) Stark effect
   (b) Population inversion
   (c) Space quantization
   (d) Bohr's correspondence principle
   (e) Einstein's coefficients
   (f) Vector atom model

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Total No. of Printed Pages—5

5 SEM TDC PHY M 4

2016

(November)

PHYSICS

(Major)

Course : 504

(Electronics)

Full Marks : 60

Pass Marks : 24 (Backlog) / 18 (2014 onwards)

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer the following as directed : 1×6=6

(a) The ratio of $I_{r.m.s.}$ and $I_{d.c.}$ of a full-wave rectifier is

(i) 0·48

(ii) 1·11

(iii) 1·21

(iv) 1·57

(Choose the correct answer)

(b) The probability of occupancy of the Fermi level at room temperature is

(i) 100%

(ii) 0%

(iii) 50%

(iv) 75%

(Choose the correct answer)

(c) Which of the following specifications is not correct for a common-collector amplifier?

(i) High-input impedance

(ii) Low-output impedance

(iii) High-voltage gain

(iv) High-current gain

(Choose the correct answer)

(d) What is the maximum theoretical efficiency of a class B push-pull transistor power amplifier?

(e) Crystal oscillators are superior to *L-C* oscillators mainly because of their

(i) small crystal size

(ii) wide frequency range

(iii) high value of *Q*

(iv) better frequency stability

(Choose the correct answer)

(f) What is the minimum number of gates required to implement the logic operation $X + \bar{X}Y$?

2. Answer the following questions : 2×6=12

(a) Intrinsic resistivity of silicon at 27 °C is $2 \cdot 8 \times 10^3$ Ω-m. If the hole and electron mobilities are 0·18 $m^2V^{-1}s^{-1}$ and 0·38 $m^2V^{-1}s^{-1}$, calculate the intrinsic carrier density of silicon.

(b) What is meant by mobility of a carrier? How does it depend upon temperature and doping concentration?

(c) Distinguish between class A and class B amplifiers.

(d) An amplifier with negative feedback gives an output of 12·5 V with an input of 1·5 V. If the feedback is removed, the same output can be obtained for an input of 0·25 V. Calculate the feedback fraction.

(e) Show how an OP-AMP can be used as an integrator.

(f) Simplify the following Boolean equation :

$$X = (A + \bar{B})(B + C)B$$

3. *(a)* Why do the energy levels of an atom become energy bands in a solid? Sketch the energy band picture for *(i)* an intrinsic, *(ii)* an *n*-type and *(iii)* a *p*-type semiconductor indicating the positions of the Fermi level, the donor or the acceptor levels. Distinguish between drift current and diffusion current in a semiconductor. 2+3+2=7

(b) Explain the terms 'barrier potential' and 'depletion region' as applied to a *p-n* junction. Plot and explain the *I-V* characteristic of a junction diode. Also write an expression for diode current. 3+3+1=7

Or

Discuss the two mechanisms of junction breakdown. Draw the circuit diagram of a d.c. power supply and explain the action of Zener diode as voltage regulator. 3+1+3=7

4. *(a)* What is non-linear distortion? Mention any two methods of minimizing it. 1+2=3

(b) What is transistor biasing? Discuss the base resistor method of biasing. What are its advantages and disadvantages? 1+3+2=6

(Continued)

Or

Write down the hybrid equations of a transistor and define the h-parameters. What are the advantages of using the h-parameters? 1+3+2=6

5. *(a)* Explain the principle of operation of Wien bridge oscillator and find an expression for the frequency of oscillation. 5

(b) Discuss briefly the steps involved in fabricating a monolithic integrated circuit. 4

6. *(a)* Draw the logic diagram of a full adder. Write the Boolean expressions for sum and carry, and give its truth table. 2+1+2=5

(b) Establish that the NAND gate is a universal gate. 2

(c) Use K-map to simplify the following Boolean expression : 2

$$X = \overline{A}B + \overline{A}\,\overline{B}\,\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C}$$

(d) Draw a logic diagram for implementation of $Y = A\overline{B} + B\overline{A}$. 1

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