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5 SEM TDC DSE MTH (CBCS)
1.1/1.2/1.3 (H)

2021

(Held in January/February, 2022)

MATHEMATICS

(Discipline Specific Elective)

(For Honours)

Paper : DSE–1

Full Marks : 80
Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : DSE–1.1

(Analytical Geometry)

1. Answer the following questions :

(a) Write the processes to sketch the parabola. 3

(b) Identify and sketch the curve

$$x = y^2 - 4y + 2$$

and also label the focus, vertex and directrix. 6

(c) Describe the graph of the curve

$$3(x+2)^2 + 4(y+1)^2 = 12$$

Also find its centre and foci. 6

Or

Describe the graph of the hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following :

(a) Fill in the blank : 1

The set of points in the plane, the sum of whose distances from two fixed points is a positive constant greater than the distance between the fixed points is _____.

(b) Write True or False : 1

A hyperbola is the set of all points in the plane that are equidistant from a fixed line and a fixed point not on the line.

(c) Suppose that an ellipse has semi-major axis a, semi-minor axis b and foci $(\pm c, 0)$. Then write the expression c in terms of a and b. 1

(d) Find the equation of the parabola that has its vertex at (1, 2) and focus at (4, 2). Also state the reflection property of parabola. 6

(e) Find the equation of the ellipse whose length of major axis is 26 and foci (±5, 0) and also sketch it. 6

Or

Find and sketch the curve of the hyperbola whose foci (6, 4) and (–4, –4) and eccentricity is 2.

3. Answer the following questions :

(a) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents hyperbola. 1

(b) Determine a rotation angle θ that will eliminate the xy-term of the conic

$$2x^2 + xy + 2y^2 + x - y = 0$$ 2

(c) Consider the equation

$$x^2 - xy + y^2 - 6 = 0$$

Rotate the coordinate axes to remove the xy-term. Then identify the type of conic represented by the equation and sketch its graph. 6

(d) Let an $x'y'$-coordinate system be obtained by rotating an xy-coordinate system through an angle $\theta = 30°$.

(i) Find the $x'y'$-coordinate of the point whose xy-coordinate is (2, 4).

(ii) Find an equation of the curve

$$2x^2 + 2\sqrt{3}xy = 3$$

in $x'y'$-coordinate. 6

Or

Identify and sketch the curve

$$153x^2 - 129xy + 97y^2 - 30x - 40y - 200 = 0$$

4. Answer the following questions :

(a) Define sphere. 1

(b) Write the equation of the sphere whose end points of the diameter is given. 1

(c) Find the equation of the sphere whose centre is (2, 3, 1) and radius is 5 units. 4

(d) Find the equation of the sphere through the origin and intersecting coordinate axes at distances a, b and c from the origin. 5

Or

A plane passes through a fixed point (a, b, c) and meets the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$

5. Answer the following questions :

(a) Define great circle. 1

(b) Write the condition that the plane

$$ax+by+cz+d=0$$

be a tangent plane to the sphere

$$x^2+y^2+z^2=r^2$$ 1

(c) Find the radius and centre of the circle

$$x^2+y^2+z^2-x-y-z-1=0,\ x+y+z=0$$ 5

(d) Find the equation of the sphere for which the circle

$$x^2+y^2+z^2+7y-2z+2=0$$

and

$$2x+3y-4z=8$$

is a great circle. 5

Or

Find the equation of the tangent planes of the sphere

$$x^2 + y^2 + z^2 - 4x - 4y - 4z + 10 = 0$$

which are parallel to the plane $x - z = 0$.

6. Answer the following questions :

(a) Write the name of cylindrical surface given by the equation

$$\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{12} = 1$$ 1

(b) Prove that the two spheres

$$x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$$

and $$x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$$

intersect each other orthogonally. 2

(c) Show that the plane

$$2x - 2y + z + 12 = 0$$

touches the sphere

$$x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$$

Also find the point of contact. 4

(d) Classify and sketch the surface

$$9x^2 + 4y^2 + z^2 = 36$$ 5

Or

Classify and sketch the surface

$$x^2 + 2z^2 - 6x - y + 10 = 0$$

Paper : DSE–1.2

(Portfolio Optimization)

1. Answer any *five* of the following as directed : 1×5=5

(a) Define investment.

(b) What is portfolio?

(c) Define risk.

(d) "Return increases with the increase in risk."

(Write True or False)

(e) Define risk-free asset.

(f) What is diversification?

2. *(a)* If an investment that costs \$250 and is worth \$350 after being held for two years, find annual holding period return (annual HPR) and annual holding period yield (annual HPY). 4

(b) Define expected return of an investment. Calculate the expected rate of return of the following economic scenarios : 1+2=3

Economic Condition	*Probability*	*Rate of Return*
Strong economy	0·15	0·20
Weak economy	0·15	–0·20
No major change in economy	0·70	0·10

(Continued)

(c) Write the measures of risk in terms of variance and standard deviation of the estimated distribution of expected returns. What is the value of variance of risk-free investment? 2+2+1=5

(d) Describe different types of risk of an investment. 5

Or

Describe the relationship between risk and return.

(e) Write a short note on mutual fund. 4

(f) Describe the investment objectives for 25-year-old investors and 65-year-old investors. 4

3. (a) What is risk aversion? 2

(b) What are the assumptions of the Markowitz's portfolio theory? 5

(c) Write the formula for the expected return for a portfolio of investments. Calculate the expected return of portfolio of risky assets given by the table : 1+2=3

Weight (w_j) (percent of portfolio)	*Expected Security Returns* (R_i)
0·20	0·10
0·30	0·11
0·30	0·12
0·20	0·13

(d) What are the variance and standard deviation of returns for an individual investment? Calculate the variance for an individual risky asset given by the following table : 2+2+3=7

Possible rate of return (R_j)	*Expected Security Return* [$E(R_j)$]	*Probability* (P_i)
0·08	0·103	0·35
0·10	0·103	0·30
0·12	0·103	0·20
0·14	0·103	0·15

Or

Describe variance and standard variation of returns for a portfolio of investments. 7

(e) Define risk-free portfolio using standard deviation of a portfolio of investments. 2

(f) Write short notes on any *two* of the following : 3×2=6

(i) Optimal portfolio

(ii) Efficient frontier

(iii) Portfolio with short sales

4. Answer any *three* of the following questions : 5×3=15

(a) Write five assumptions of capital market theory.

(b) Derive the risk-return combination equation of capital market theory.

(c) Derive the equation of the capital asset pricing model (CAPM).

(d) Determine the expected rate of return with CAPM for the following five stocks :

Stock	*Beta*
A	0·70
B	1·00
C	1·15
D	1·40
E	–0·30

where economy's RER = 0·05 and expected return on the market portfolio $E(R_M) = 0 \cdot 09$.

5. What is security market line (SML)? What are the differences between capital market line (CML) and security market line (SML)? 1+2=3

6. Suppose that during the most recent 10 years period, the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ($\overline{R_M} = 0 \cdot 14$) and the average nominal rate of return on government T-bills was 8 percent ($\overline{RFR} = 0 \cdot 08$). As administrator of a large pension fund that has been divided among three money managers during the past 10 years. Decide by calculating T values whether to renew their investment management contracts based on the following results :

Investment Manager	*Average Annual Rate of Return*	*Beta*
W	0·12	0·90
X	0·16	1·05
Y	0·18	1·20

Also plot their portfolios with security market line (SML). 7

Or

Describe sharp portfolio performance measure with example.

Paper : DSE–1.3

(Financial Mathematics)

UNIT—I

1. Answer the following questions : 1×4=4

(a) Define cash flow.

(b) Write to which greater expected return is related.

(c) If the interest rate is r, then write the price of an investment that pays A after one year.

(d) Write which entirely determines internal rate of return.

2. Answer the following questions : 2×4=8

(a) Explain the viewpoint of investment.

(b) Write about the investment and return for the situation represented by cash flow $(-1, 1 \cdot 4)$.

(c) Write the relation between present value and future value.

(d) Explain callable bond.

3. Answer any *four* of the following questions : 4×4=16

(a) Explain comparison principle.

(b) Write the main features of hedging.

(c) Write the objectives of pure investment.

(d) Show that growth under compound interest is geometric.

(e) Compute the future value of the cash flow stream (–4, 1, 2, 1).

(f) Describe effective interest rate and nominal rate.

4. Answer any *two* of the following questions : 6×2=12

(a) Find the internal rate of return by solving the equation $x^3 + x^2 + x = 2$ (use Newton-Raphson method).

(b) State and prove the main theorem of internal rate of return.

(c) Describe duration.

(d) Show that derivation of price P with respect to yield λ of a fixed income security is

$$\frac{dP}{d\lambda} = -D_m P$$

where D_m is modified duration.

UNIT—II

5. Answer the following questions : 1×4=4

(a) Define random variable.

(b) Define diversification.

(c) Write one property of feasible set.

(d) Write through which capital market line passes.

6. Answer the following questions : 2×4=8

(a) Write about short selling.

(b) Find the expected value of the number of spots on a roll of a die.

(c) Write two properties of expected value.

(d) Define covariance of two random variables x_1, x_2.

7. Answer any *four* of the following questions : 7×4=28

(a) Show that the rate of return acts like an interest rate.

(b) Find the mathematical expression for total return.

(c) Show that the variance of the return of the portfolio

$$\sigma^2 = \sum_{i,\, j=1}^{n} w_i w_j \sigma_{ij}$$

(d) State the capital asset pricing model and prove it.

(e) Define mean standard deviation diagram and show that

$$\text{var}(x) = E(x^2) - \bar{x}^2$$

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