5 SEM TDC PHYH (CBCS) C 11

2021

(Held in January/February, 2022)

PHYSICS

(Core)

Paper : C–11

(Quantum Mechanics and Applications)

(Theory)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer from the following : 1×5=5

(a) A wave function $\psi(\vec{r}, t)$ is admissible, if

(i) ψ is single-valued and finite

(ii) ψ is single-valued

(iii) ψ is finite

(iv) ψ is finite and multi-valued

(b) Stationary states are those for which the probability density ρ is

(i) time-dependent

(ii) time-independent

(iii) space-dependent

(iv) space-independent

(c) The zero-point energy of the simple harmonic oscillator is

(i) ∞

(ii) $\frac{1}{2}\hbar\omega$

(iii) $\frac{3}{2}\hbar\omega$

(iv) 0

(d) The electron in a hydrogen atom moves in a potential which is regarded as

(i) asymmetric

(ii) spherically symmetric

(iii) coulombian

(iv) None of the above

(e) Possible values of the Z-component of spin angular momentum are given by

(i) $\pm h$

(ii) $\pm \frac{\hbar}{2}$

(iii) $\pm \hbar$

(iv) $\pm 2\hbar$

2. Prove that the relation $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$, where $\vec{J}$ is the probability current density and ρ is the probability density. 3

Or

What do you understand by normalized wave function? Find the normalization constant of the particle described by the Gaussian wave packet wave function $\psi(x) = Ae^{-\alpha^2 \frac{x^2}{2}} e^{ikx}$, (given $\int_{-\infty}^{\infty} e^{-\alpha^2 x^2} dx = \frac{\sqrt{\pi}}{\alpha}$).

3. Show that position and linear momentum operators do not commute, i.e., $[\hat{x}, \hat{p}_x] = i\hbar$. 2

Or

Find the expectation value of momentum for the wave function $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$ in the region $0 < x < L$.

4. Plot graphically the Gaussian wave packet given by the equation $\psi(x) = \frac{1}{\sqrt{\sigma\sqrt{\pi}}} e^{-x^2/2\sigma^2}$ where $\sigma^2 = \hbar c$ and explain its properties. 4

Or

State and prove Heisenberg's uncertainty principle for wave packets. If the product of uncertainties in position and momentum is minimum, find the form of the function.

5. Find out momentum wave function expression for a free particle in three dimensions. 3

6. Show that the energy of a particle trapped in a one-dimensional box of length a is

$$E_n = \frac{n^2 \hbar^2 \pi^2}{2ma^2}$$ 3

7. Write down the Schrödinger equation of a one-dimensional harmonic oscillator. What is the energy of this oscillator when it is in the eigenstate associated with the quantum number n? Discuss the significance of its zero-point energy. 5

Or

Write the general and normalized wave function of a harmonic oscillator. State the first two normalized wave functions of the oscillator.

8. If the expectation values of the square of the displacement harmonic oscillator is

$$< x^2 >= \left(n+\frac{1}{2}\right)\hbar / m\omega$$

what is the expectation value of the potential energy? 3

9. Write down the time-independent Schrödinger equation for the motion of the electron in hydrogen atom, assuming the proton to be at rest.

Given

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial\phi^2}$$

Separate the Schrödinger equation into one radial and two angular parts. 1+4=5

Or

What is azimuthal component of Schrödinger's wave equation of hydrogen atom? Obtain its solution and normalized eigenfunction. What is the significance of the quantum number predicted by it? 1+3+1=5

10. The radial part of wave function for hydrogen in the ground state is given by

$$R = \frac{2}{a_0^{3/2}} e^{-\frac{r}{a_0}}$$

Find the expression for ground-state energy of hydrogen atom $(n = 1, l = 0)$. 3

11. Describe the Stern-Gerlach experiment for verification of space quantization. 5

12. What is normal Zeeman effect? On the basis of quantum theory, explain the effects of magnetic field on energy levels of an atom. 4

13. Discuss the *L-S* coupling scheme. 4

14. How can the states of an atom be represented in spectral notation? 4

Or

What is meant by fine structure of spectral lines? Describe how the spin orbit coupling explains the fine structure of alkaline spectra. 1+3=4

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