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1 SEM TDC PHYH (CBCS) C 1

2021

(Held in January/February, 2022)

PHYSICS

(Core)

Paper : C–1

(Mathematical Physics—I)

Full Marks : 53

Pass Marks : 21

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Choose the correct answer : 1×5=5

(a) The partial derivative of $ye^{2x}+2xy^2$ is

(i) $2(ye^{2x}+xy^2)$

(ii) $2(ye^{2x}+y^2)$

(iii) $(ye^{2x}+2y^2)$

(iv) None of the above

(b) The degree and order of the differential equation

$$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^2 + 2y = e^{3x}$$

are

(i) 2 and 2

(ii) 2 and 1

(iii) 1 and 2

(iv) None of the above

(c) If $\vec{A}$ is an irrotational vector, then

(i) $\vec{\nabla} \cdot \vec{A} = 1$

(ii) $\vec{\nabla} \times \vec{A} = 0$

(iii) $\vec{\nabla}\vec{A} = 0$

(iv) None of the above

(d) By Gauss divergence theorem, $\int_V \vec{\nabla} \cdot \vec{A} dV$ equals to

(i) $\int_S \vec{A} \cdot d\vec{S}$

(ii) $\oint_C \vec{A} \cdot d\vec{r}$

(iii) $\oint_C \vec{A} \cdot d\vec{S}$

(iv) None of the above

(e) A normal to the surface $\phi(x, y, z) = c$ is given by

(i) $\vec{\nabla} \cdot \phi$

(ii) $\vec{\nabla} \times \phi$

(iii) $\vec{\nabla}\phi$

(iv) None of the above

2. Answer the following questions : 2×5=10

(a) Show that $\lim_{x \to 0} \sin\frac{1}{x}$ does not exist.

(b) For what values of a, $\vec{A}$ and $\vec{B}$ are perpendicular if $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$?

(c) What is a Wronskian? How is it used to find the linear dependence of two functions?

(d) Show that $\vec{B}$ is perpendicular to $\vec{A}$, if $|\vec{B}| \neq 0$ and $\vec{B} = \frac{d\vec{A}}{dt}$.

(e) Evaluate using the property of Dirac delta function :

$$\int_{-\infty}^{+\infty} x\delta(x-4)dx$$

3. Answer any *five* questions from the following : 4×5=20

(a) What do you mean by linearly dependent and linearly independent solutions of a homogeneous equation? If $y_1(x) = \sin 3x$ and $y_2(x) = \cos 3x$ are two solutions of $y'' + 9y = 0$, then show that $y_1(x)$ and $y_2(x)$ are linearly independent solutions. 1+3=4

(b) If $z(x+y) = x^2 + y^2$, then show that

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$ 4

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Hence find the solution for

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$$ 3+1=4

(d) What is directional derivative? Find the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at (1, 1, −1) in the direction $2\hat{i} + \hat{j} - \hat{k}$. 1+3=4

(e) State Bayes' theorem of probability. 6 cards are drawn from a pack of 52 cards. What is the probability that 3 will be red and 3 black? 1+3=4

(f) State Green's theorem in a plane. Starting from Green's theorem, show that the area bounded by a closed curve is given by

$$\frac{1}{2}\oint_C (x\,dy - y\,dx)$$ 1+3=4

4. Answer any *three* questions from the following : 6×3=18

(a) What are complementary function and particular integral of a differential equation? Solve the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$$

if $y(0) = 0$ and $y'(0) = \frac{1}{2}$. 1+5=6

(b) Define line integral and surface integral. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along a curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. 2+4=6

(Turn Over)

(c) Show that $F = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential. Also find the work done in moving an object from (1, −2, 1) to (3, 1, 4). 2+2+2=6

(d) What are curvilinear coordinates? Describe the term 'scale factor' in curvilinear coordinates. Derive the expression for divergence of a vector in curvilinear coordinates. Hence write its expression in spherical polar coordinates. 1+2+3=6

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